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Transient Thermal Response of Ablating Bodies

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A numerical study of transient thermal response of a blunt-nosed axisymmetric body made of Teflon is presented using a two-layer thermal model. It is shown that phase change and transverse heat conduction have a considerable effect on the internal temperature field. Comparison of the numerical results with experimental data shows that the single-layer thermal model does not predict the real feature of the thermal field, whereas the results of the two-layer thermal model agree reasonably well with the experiment.

Nomenclature

B	= mass-addition parameter, $\dot{m}(H_{st}-H_w)/q_{o,B=0}$
h_m	= energy required to phase transition
h_v	= energy required to ablate unit mass of body
H	= total enthalpy
k	= thermal conductivity
L	= thickness of the solid layer in x direction
L_0	= initial body length
$\dot{m}$	= ablation rate
M	= Mach number
q_o	= heat flux across outside surface (Fig. 2)
q_s	= heat flux across inside surface (Fig. 2)
s	= recession depth in axial direction
t	= time
T	= temperature
v	= velocity normal to the surface
(x,r)	= cylindrical coordinates (Fig. 1)
x_i	= axial position of the original surface (Fig. 1)
X	= transformed geometric variable defined by Eq. (1)
α	= thermal diffusivity
δ_i	= angle between local interface tangent in meridian plane and axis of symmetry
δ_s	= angle between local surface tangent in meridian plane and axis of symmetry (Fig. 2)
ζ	= distance measured along a meridian line from the stagnation point (Fig. 2)
θ	= thickness of the gel layer in axial direction (Fig. 1)
ξ	= transformed geometric variable defined by Eq. (2)
ρ	= material density of the ablating body
ψ	= ratio of the heating rate with mass addition to the heating rate without mass addition

Subscripts

$B=0$	= conditions for no mass addition
st	= stagnation condition in freestream
1	= conditions in gel layer
2	= conditions in solid layer

Introduction

FOR high-speed entry of space vehicles into atmospheric environments, ablation is a practical method for alleviating severe aerodynamic heating. Several studies[1-4] have been undertaken on steady ablation. However, ablation is a very complicated phenomenon in which a chemicophysical process is associated with an aerodynamic one that involves changes in body shape with time. Therefore, it seems realistic to consider that ablation is an unsteady phenomenon. In the design of an ablative heat protection system, since the ultimate purpose of the heat shield is to keep the internal temperature of the space vehicle at a safe level during entry, the transient heat conduction characteristics of the ablator may be one of the significant factors in the selection of the material and its thickness.

Studies that have been undertaken on quasisteady ablation[5-10] offer some useful information about the transient thermal response of the ablators. On the other hand, in the numerical studies proposed by Friedman and MacFarland[11] for an ablating thrust chamber wall of a rocket engine and by Popper et al.[12] and Tompkins et al.[13] for ablating blunt-nosed axisymmetric bodies, the effect of changes in body shape is taken into account.

As to ablators made of high molecular compounds such as Teflon (polytetrafluoroethylene), there exists the second-order transition temperature at which physical properties change abruptly. This transition may be interpreted as melting. However, the melted Teflon has considerably high viscosity, so that its fluidity may be essentially negligible, although it might be a liquid. In this sense, the melt is called "gel."

Most of the existing theories associated with the ablation of Teflon have been developed using a single-layer thermal model, where the existence of the gel layer is ignored. However, there are a few previous analytical studies[14-16] of the transient thermal response, undertaken on the two-layer thermal model, which consists of a gel layer and a solid material. They are essentially one-dimensional and give meager information available for practical application.

This investigation presents a numerical study of the transient thermal response of an ablating body made of Teflon. With emphasis on the effects of the gel layer and the transverse diffusion of heat on the transient temperature distribution in the ablating bodies, the formulation is made using the two-layer thermal model under the assumptions that the boundary-layer flow is quasisteady and that the vaporization at the ablating surface is in equilibrium. Numerical calculations are carried out for a blunt-nosed axisymmetric body.

Basic Equations

The formulation is made using the two-layer thermal model along with several basic assumptions, which are summarized as follows:

1) Thermal properties are constant in each layer (see Table 1).

2) Fluidity of the gel is neglected.

3) Inviscid flow outside the boundary layer is given by the modified Newtonian theory.

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4) Boundary-layer flow is quasisteady (see Ref. 17).
5) Thermal deformation of the body is neglected.

Strictly speaking, the thermal expansion of Teflon is important enough so that the thermal deformation of the body shape should be taken into account.[18] However, it may be deduced that this effect will not violate the essential characteristics of the transient thermal field inside the body except for the ablation rate, because the displacement of the point under observation still remains small relative to the size of the body.

For heat-conduction problems including surface recession, it is convenient to use a coordinate system that is fixed on the moving surface. Figure 1 is an illustration of the transformed coordinate system used in the present analysis. The cylindrical coordinates (x,r) are transformed into body-fixed coordinates (X,r) by the following expression:

$$X=\{[x-(x_i+s)]/\theta\}-1 \tag{1}$$

where $x_i(r)$ denotes the original surface at $t=0$, $s(r,t)$ the axial distance of the surface recession due to the ablation, and $\theta(r,t)$ indicates the instantaneous thickness of the gel layer measured in the direction of x axis. For the solid layer, the X coordinate can be simplified further by introducing a transformation

$$\xi=X/(L/\theta) \tag{2}$$

With these transformations, the heat-conduction equation can be expressed for the gel layer $(-1\leq X\leq 0)$ as

$$\frac{\partial T_1}{\partial t}=\alpha_1\left(\frac{\partial^2 T_1}{\partial r^2}+\frac{1}{r}\frac{\partial T_1}{\partial r}\right)+\frac{\alpha_1}{\theta^2}\frac{\partial^2 T_1}{\partial X^2}(1+A_1^2)$$
$$-\frac{2\alpha_1 A_1}{\theta}\frac{\partial^2 T_1}{\partial X\partial r}+\frac{1}{\theta}\frac{\partial T_1}{\partial X}\left[\frac{\partial s}{\partial t}+(X+1)\frac{\partial \theta}{\partial t}-\frac{\alpha_1}{r}A_1\right.$$
$$\left.-\alpha_1\left(\frac{\partial A_1}{\partial r}-\frac{2A_1}{\theta}\frac{\partial\theta}{\partial r}\right)\right] \tag{3}$$

where

$$A_1=\frac{\mathrm{d}x_i}{\mathrm{d}r}+\frac{\partial s}{\partial r}+(X+1)\frac{\partial\theta}{\partial r}$$

and for the solid layer $(0\leq\xi\leq 1)$ as

$$\frac{\partial T_2}{\partial t}=\alpha_2\left(\frac{\partial^2 T_2}{\partial r^2}+\frac{1}{r}\frac{\partial T_2}{\partial r}\right)+\alpha_2\frac{\partial^2 T_2}{\partial \xi^2}\left\{\frac{1}{L^2}+\left[\frac{L\xi}{\theta}\frac{\partial}{\partial r}\left(\frac{\theta}{L}\right)\right.\right.$$
$$\left.\left.-\frac{A_2}{L}\right]^2\right\}+\frac{\partial T_2}{\partial \xi}\left[\alpha_2\xi\left(\frac{1}{\theta}\frac{\partial^2\theta}{\partial r^2}-\frac{1}{L}\frac{\partial^2 L}{\partial r^2}\right)+\frac{1-\xi}{L}\left(\frac{\partial s}{\partial t}+\frac{\partial\theta}{\partial t}\right)\right.$$
$$+\frac{\alpha_2}{\theta}\left(\frac{L\xi}{r}-2A_2-2\xi\frac{\partial L}{\partial r}-\frac{L\xi}{\theta}\frac{\partial\theta}{\partial r}\right)\frac{\partial}{\partial r}\left(\frac{\theta}{L}\right)+\frac{\alpha_2}{L}$$
$$\left.\times\left(\frac{2A_2}{\theta}\frac{\partial\theta}{\partial r}-\frac{A_2}{r}-\frac{\partial A_2}{\partial r}\right)\right]+2\alpha_2\frac{\partial^2 T_2}{\partial\xi\partial r}\left[\frac{L\xi}{\theta}\frac{\partial}{\partial r}\left(\frac{\theta}{L}\right)-\frac{A_2}{L}\right] \tag{4}$$

where

$$A_2=\frac{\mathrm{d}x_i}{\mathrm{d}r}+\frac{\partial s}{\partial r}+\left(\frac{L\xi}{\theta}+1\right)\frac{\partial\theta}{\partial r}$$

$$L=L_0-(x_i+s+\theta)$$

where $T(X,r,t)$, $L_0(r)$, and α denote temperature, initial body thickness, and thermal diffusivity, respectively, and the subscripts 1 and 2 indicate the conditions in the gel and the solid phases, respectively.

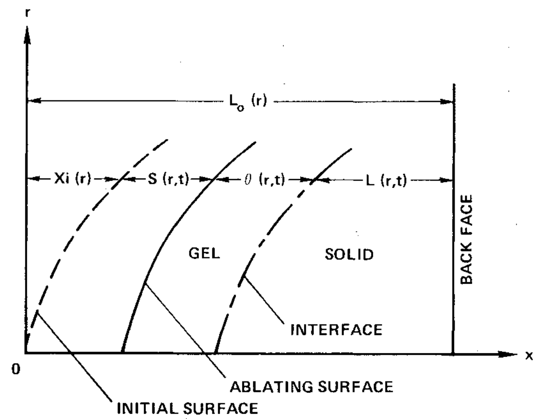

Fig. 1 Coordinate system.

Initial Conditions and Boundary Conditions

The initial temperature of the body is uniform, so that the ablation rate is negligible:

$$T(X,r,0)=\text{const}=15°\text{C},\quad \dot{m}(r,0)=0 \tag{5}$$

where $\dot{m}(r,t)$ is the ablation rate.

Figure 2 illustrates a control surface with unit area on the ablating surface $(X=-1)$, where q_o and q_s denote heating rate from the boundary layer and heat flux conducting inside the body, respectively. The heat-transfer balance normal to the control surface may be given as

$$q_s=q_o-\dot{m}h_v,\quad \dot{m}=\rho_1\frac{\partial s}{\partial t}\sin\delta_s \tag{6}$$

where h_v and δ denote the energy required for vaporization of the unit mass of the gel layer and the angle that the local surface makes with the axis of symmetry, respectively. The subscript s indicates the conditions at the ablating surface. Another geometrical relation is given as

$$\cot\delta_s=A_1-\frac{\mathrm{d}L_0}{\mathrm{d}r} \tag{7}$$

By use of this equation, together with the expression

$$q_s=-\left(k_1\frac{\partial T_1}{\partial x}\right)_s\sin\delta_s+\left(k_1\frac{\partial T_1}{\partial r}\right)_s\cos\delta_s \tag{8}$$

the equation of heat flux balance at the surface, Eq. (6), can be reduced to

$$(q_o-\dot{m}h_v)\sin\delta_s=\left(k_1\frac{\partial T_1}{\partial r}\right)_s\sin\delta_s\cos\delta_s$$
$$-\frac{1}{\theta}\left(k_1\frac{\partial T_1}{\partial X}\right)_s\left(1+\frac{\mathrm{d}L_0}{\mathrm{d}r}\sin\delta_s\cos\delta_s\right) \tag{9}$$

Table 1 Physical properties of Teflon

Value	Reference
$k_1=2.00\times10^{-4}$ cal-cm^{-1}-s^{-1}-deg^{-1}	14, 17
$k_2=6.00\times10^{-4}$ cal-cm^{-1}-s^{-1}-deg^{-1}	14, 17
$\alpha_1=4.02\times10^{-4}$ cm^2-s^{-1}	17
$\alpha_2=1.19\times10^{-3}$ cm^2-s^{-1}	17
$\rho_1=1.72$ g-cm^{-3}	14, 15, 17
$\rho_2=2.10$ g-cm^{-3}	14, 15, 17
$h_v=370$ cal-g^{-1}	22
$h_m=14$ cal-g^{-1}	22
$T_m=327°$C	22

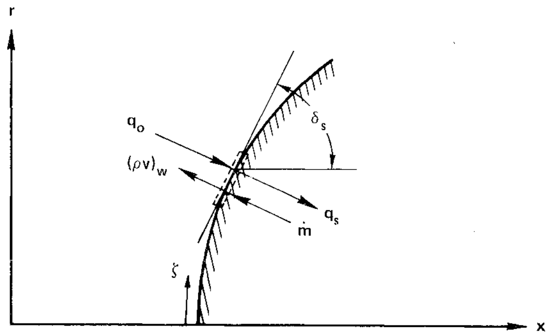

Fig. 2 Control surface of heat flux balance.

where k is thermal conductivity.

At the interface between the gel and the solid phases ($X=0$), the temperature must be continuous; that is

$$T_1 = T_2 = 327°\text{C} \tag{10}$$

and the heat flux balance can be obtained in the form

$$-\left(\frac{k_1}{\theta}\frac{\partial T_1}{\partial X}\right)_i \sin\delta_i + k_1\left(\frac{\partial T_1}{\partial r} - \frac{A_1}{\theta}\frac{\partial T_1}{\partial X}\right)_i \cos\delta_i$$
$$-\rho_2 h_m\left(\frac{\partial s}{\partial t} + \frac{\partial \theta}{\partial t}\right)\sin\delta_i$$
$$= -\left(\frac{k_2}{\theta}\frac{\partial T_2}{\partial X}\right)_i \sin\delta_i + k_2\left(\frac{\partial T_2}{\partial r} - \frac{A_2}{\theta}\frac{\partial T_2}{\partial X}\right)_i \cos\delta_i \tag{11}$$

where h_m denotes the latent heat for phase transition from the solid to the gel, and the subscript i indicates the conditions at the interface. It must be noted that the geometrical relation to be satisfied at the interface has been employed to obtain Eq. (11), which is given as

$$\cot\delta_i = A_2 - \frac{dL_0}{dr} \tag{12}$$

It is assumed that the back surface is thermally insulated; that is,

$$\left(\frac{\partial T_2}{\partial \xi}\right)_{\xi=1} = 0 \tag{13}$$

In the axisymmetric case, an additional condition of symmetrical temperature distribution with respect to the axis of symmetry is required; that is,

$$\left(\frac{\partial T}{\partial r}\right)_{r=0} = 0 \tag{14}$$

Other Relations Associated with the Ablating Flowfield

Since the local ablation rate is dependent on time and location, the body shape changes with time. This leads to changes in flow around the body and consequently in local heat-transfer rates. These effects should have been taken into account by evaluating the boundary-layer growth properly. However, from the quasisteady assumption of the boundary-layer flow, they may be evaluated reasonably by use of the steady heating rate q_o proposed by Lees,[19] if it is evaluated under the instantaneous boundary conditions.

On the other hand, when ablation occurs, a small amount of mass is injected into the boundary layer, so that the heat transfer to the wall will be reduced. Therefore, q_o must be modified by taking the effect of the mass injection into account. As to the stagnation flow, it has been shown that the effect of the mass addition can be formulated as a function of the mass addition parameter B only. Marvin and Pope[3] reviewed the data for Teflon and presented an empirical relation, which is expressed as

$$\Psi(B) = \frac{q_{o,B}}{q_{o,B=0}} = 1 - 0.72\left(\frac{Ma}{Mt}\right)^{0.25} B + 0.13\left(\frac{Ma}{Mt}\right)^{0.5} B^2 \tag{15}$$

where Ma and Mt are molecular weights of the air and Teflon, respectively.

It must be noted that the injected mass will influence the boundary-layer flow downstream. However, because there does not seem to exist any conventional method available for estimating the local heating rate with mass injection over the entire surface, it is assumed in the present investigation that the modification given by Eq. (15) is applicable everywhere on the ablating surface. A rough evaluation reveals that the ablation rate obtainable within the range of the present calculation is of the order of 10^{-3} g-cm^{-2}-s^{-1} for Teflon. From this fact together with the results presented by Swann and South[20] and Rubesin and Inouye,[21] it appears that the present assumption is valid.

Since the ablation rate is essentially a kind of chemicophysical reaction rate, it can be expressed in terms of two thermodynamic variables of state, such as temperature and pressure, under the assumption of equilibrium vaporization. In the case of Teflon, Wentink[22] pointed out that the ablation rate is almost independent of pressure when $p < 1$ atm and $T_w < 800°$C. This fact seems to be supported by the other experiments.[23,24] Therefore, it is assumed that the ablation rate is a function of the surface temperature only and is given by Rashis and Hopko.[25]

Numerical Calculation

Examination of the basic equations, Eqs. (3) and (4), reveals that, along the axis of symmetry, there exists an apparent singularity in the terms including $1/r$. However, it can be eliminated numerically in such a way that the numerator tends to vanish as the denominator approaches zero.

In the present investigation, the basic equations were integrated numerically by use of the alternating direction implicit (ADI) method,[26] where all derivatives in the equations were evaluated to second-order accuracy. However, it must be noted that the successive pairs of the integration technique must have, in general, the same time step to keep the calculation stable. The numerical computation was carried out in the following procedures. Since the prescribed initial temperature distribution $T(X,r,0)$ is lower than the phase transition temperature T_m, the heat-conduction equation is solved first by using the single-layer thermal model up to the time that the phase transition temperature is reached at a point on the surface. With the temperature distribution at this time as an initial condition, the calculation is continued further by use of the two-layer thermal model over the specified time period.

Results and Discussion

The surface temperature and the thickness of the gel layer on the axis of symmetry are shown in Fig. 3; the overshoot in each curve may be caused by the roundoff error in numerical calcualtion. The surface temperature rises very rapidly up to the phase transition temperature; beyond that temperature, it rises even more abruptly. The surface temperature reaches a constant value in a short time and then remains almost unchanged, suggesting that the ablation rate is steady.

On the other hand, the gel layer, which begins to grow abruptly at the time the transition temperature is approached, still continues to increase its thickness even after the surface temperature has reached the steady values. This implicitly suggests that the temperature in the solid phase is still varying with time and that the assumption on the steady ablation is invalid.

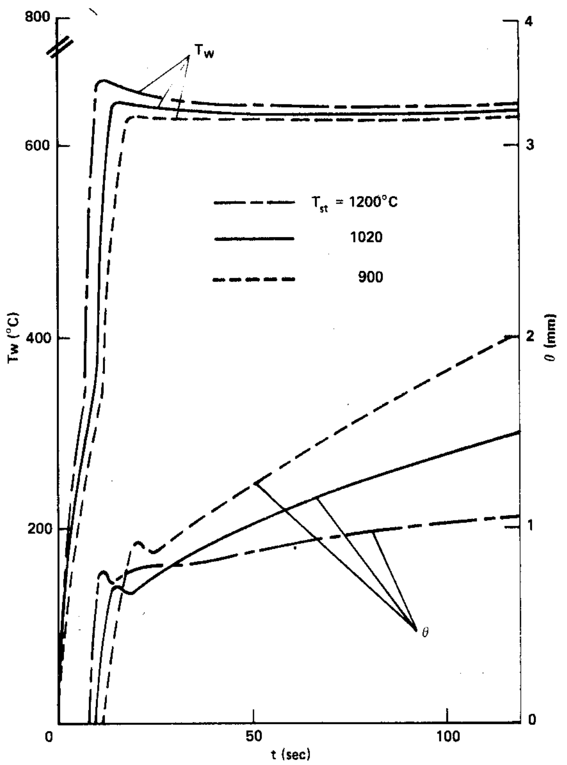

Fig. 3 Surface temperature and thickness of gel layer at stagnation point ($M = 5.74$, $p_{st} = 1$ atm, $r = 0$ cm).

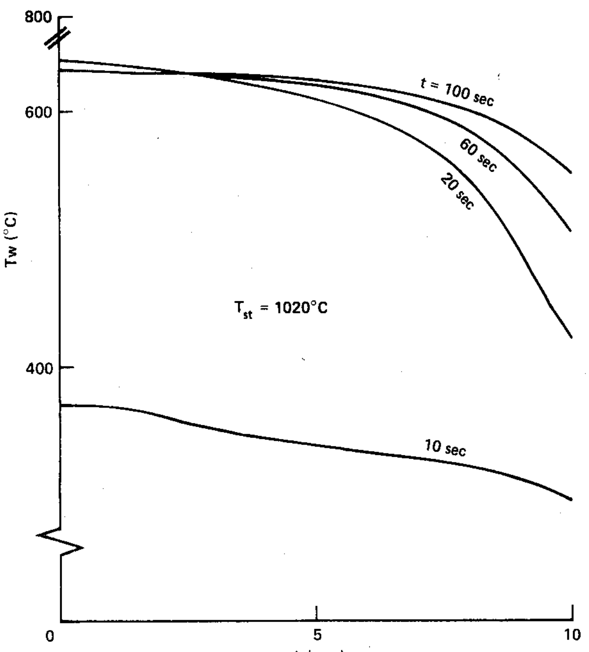

Fig. 4 Distribution of instantaneous surface temperature ($M = 5.74$, $p_{st} = 1$ atm, $T_{st} = 1020°$C).

A typical example of the instantaneous temperature distribution along the ablating surface is presented in Fig. 4, where ζ denotes the distance measured along the meridian line from the stagnation point. This clearly indicates that the surface temperature far downstream from the stagnation point is still rising even at $t = 100$ s. This implies that the shape of the body is changing at that time.

To permit examination of successive changes in the internal temperature distribution with time, several typical examples of the instantaneous contour of the isothermal surface in a meridian plane are shown in Fig. 5. The isothermal contours

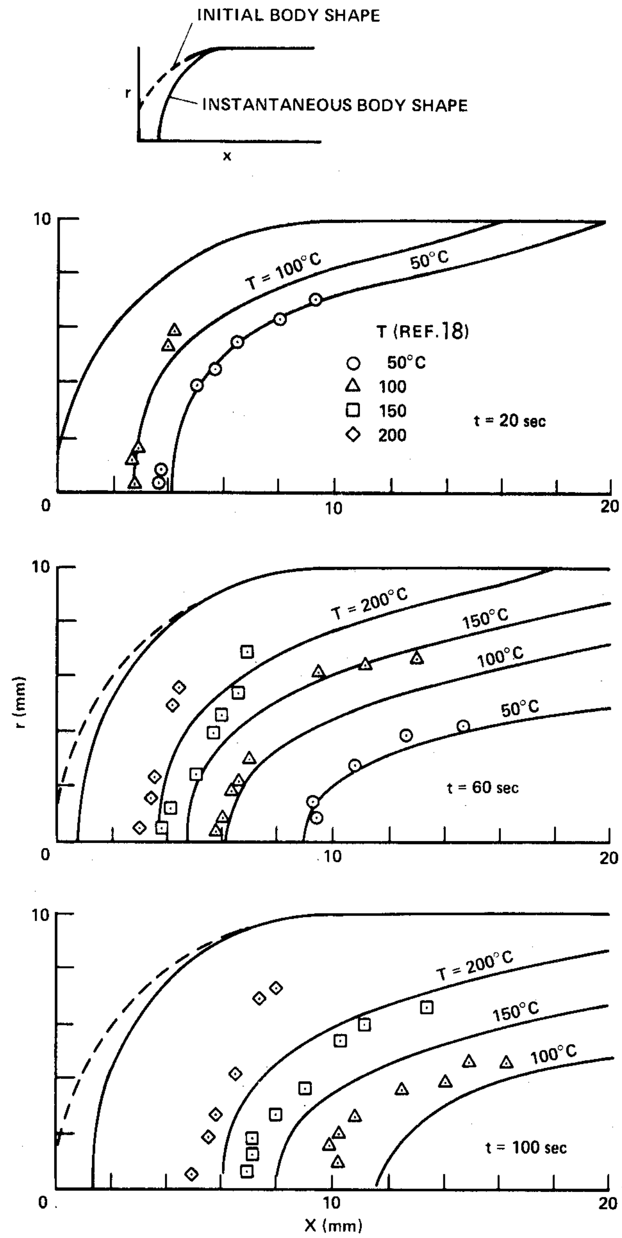

Fig. 5 Instantaneous contour of isothermal surface ($M = 5.74$, $p_{st} = 1$ atm, $T_{st} = 1020°$C).

are similar at each time. However, in the region far from the axis of symmetry, the local slope of the isothermal contour at a fixed temperature level decreases considerably with time; this reveals the interesting fact that the transverse heat conduction is a significant factor in increasing the internal temperature in regions other than on the axis of symmetry. The present results agree well with the experiment[18] not only qualitatively but also quantitatively in the early time period. Although the quantitative difference between the numerical results and the experiment may increase with time because of the accumulation of error in the calculation, the qualitative agreement seems to be preserved fairly well.

For the purpose of demonstrating the effect of the gel layer on the transient thermal field, the numerical computation was carried out further using the single-layer thermal model; the results of instantaneous temperature distribution along the axis of symmetry are presented in Fig. 6 together with those of the two-layer thermal model and the experimental data. Although the solutions for the single-layer thermal model may be nearly similar to those for the two-layer thermal model initially, the discrepancy between the two models becomes increasingly significant with time. It seems that this

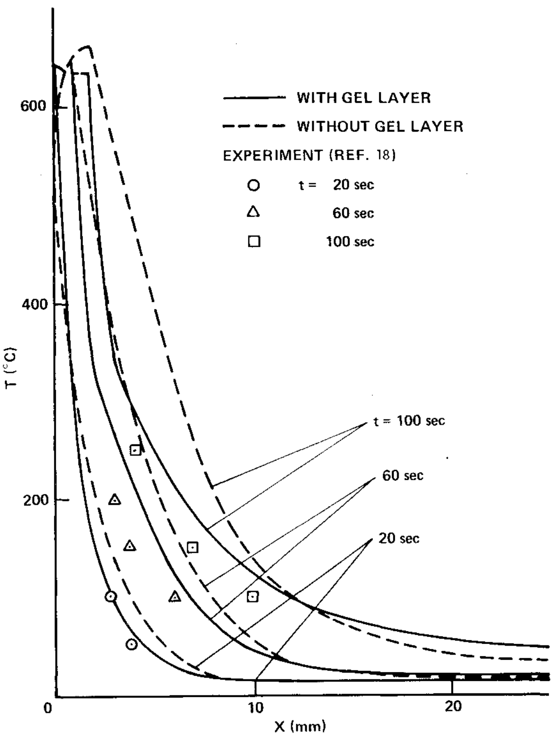

Fig. 6 Axial distribution of instantaneous temperature ($M = 5.74$, $p_{st} = 1$ atm, $T_{st} = 1020°C$).

is caused by the difference of the thermal diffusivity between the gel layer and the solid layer. The agreement with the experiments is better for the two-layer thermal model. The analysis based on the single-layer thermal model cannot give a correct quantitative prediction of the real thermal field in the ablator, and even the qualitative agreement is unsatisfactory.

Conclusions

With emphasis on the second-order phase transition, a numerical investigation of the transient thermal response of Teflon has been developed using the two-layer thermal model. The local ablation rate tends to reach a constant value in a short time, whereas the temperature in the solid material changes with time because of the duplicate effects of the growth of the gel layer and the transverse heat conduction. This result is quite consistent with the experiment.

The thickness of the gel layer decreases with increasing surface temperature. This, in turn, leads to the deduction that the smaller thickness of the gel layer results in a steeper descent of the temperature across the layer and, consequently, in an increase of the heat-shield effectiveness of the ablator. Finally, it is suggested that any theoretical study of the transient thermal response of an ablating body made of high-molecular weight compounds such as Teflon should be undertaken on the basis of the multilayer thermal model.

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